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COMMENT

Estimates of the site percolation probability exponents for some directed lattices

K De'Bell† and J W Essam‡

† Department of Physics, Dalhousie University, Halifax, Nova Scotia, Canada B3H 3J5
‡ Mathematics Department, Westfield College, University of London, London NW3 7ST, England

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Abstract. Series expansion analysis has been used to obtain estimates of the percolation probability exponent, β, for directed site percolation on the square, triangular, simple cubic and body centred cubic lattices. These estimates are consistent with those obtained by Blease for the corresponding bond problems and the results for site and bond problems on all lattices considered are summarised by $\beta = 0.28 \pm 0.01$ $(d = 2)$ and $\beta = 0.59 \pm 0.02$ $(d = 3)$.

Recently reported estimates of the mean cluster size exponent γ and perpendicular and parallel length exponents $\nu_\perp$ and $\nu_\|$ for directed percolation (De'Bell and Essam 1983a, b) support the validity of the hyperscaling relation

$$2\beta = (d-1)\nu_\perp + \nu_\| - \gamma \tag{1}$$

for lattices of dimensionality $d = 2$ and 3. Estimates of the percolation probability exponent β by the use of series expansion methods were obtained by Blease (1977) but were restricted to the bond percolation problems. Here we present estimates of β, obtained by analysing the initial terms of the series expansions of the percolation probability, for site percolation on the directed square, triangular, simple cubic and body centred cubic lattices.

In all cases, all parallel bonds in a lattice are directed in the same sense so that fluid flow is always positive along some chosen axis. Sites of the lattice are present with probability p and absent with probability q $(=1-p)$. The percolation probability may be expressed as

$$P(q) = 1 - \sum_{c \in \mathscr{C}_0} p^{s(c)} q^{t(c)}, \tag{2}$$

where $\mathscr{C}_0$ is the set of finite clusters with source at the origin and $s(c)$ and $t(c)$ denote the number of sites (excluding the origin) in the cluster c and its perimeter respectively (Domb 1959). Clusters with a perimeter size less than or equal to a given maximum were enumerated by a lattice animal generating program, and the resulting series expansions for the percolation probability are presented in table 1.

Padé approximants to the series $\partial \ln P(p)/\partial p$ for each of the lattices considered were formed and pole residue curves plotted. Estimates of the critical value of $q = q_c$ have been previously obtained by analysing the low-density mean cluster size series (De'Bell and Essam 1983a, b). The corresponding estimates of β were read from the

Table 1. Tabulation of the percolation probability $P(q) = 1 - \Sigma_r a_r q^r$ for site percolation on the directed lattices.

r \ Lattice	SQ	T	SC	BCC
2	1	0	0	0
3	3	1	1	0
4	8	2	0	1
5	21	5	3	0
6	56	10	1	0
7	154	20	0	4
8	434	41	21	0
9	1252	86	−34	−1
10	3675	182	101	0
11		393	−249	28
12		853	921	−1
13			−2524	−118
14			5613	294
15			−8914	−184
16			6206	−441
17				1486
18				−273
19				−6464
20				8969

pole residue plots in the usual way and are presented in table 2. We also give estimates for the bond problem derived from the series of Blease (1977) together with our revised critical points (De'Bell and Essam 1983a, b).

In two dimensions there is excellent agreement between the values for all four problems. The larger error bar for the square lattice bond problem was assigned on

Table 2. Estimates of β for percolation on the directed lattices.

(*a*) *Site problem.*

Lattice	SQ	T	SC	BCC
q_c†	0.2945 ± 0.0005	0.4051 ± 0.0004	0.566 ± 0.004	0.656 ± 0.004
β	0.2725 ± 0.0015 $+6\Delta q_c$	0.2835 ± 0.0010 $+9\Delta q_c$	0.568 ± 0.004 $+13\Delta q_c$	0.578 ± 0.002 $+12\Delta q_c$

(*b*) *Bond problem* (results obtained from a reanalysis of the series of Blease (1977)).

q_c†	0.3554 ± 0.0002	0.5223 ± 0.0005	0.618 ± 0.001	0.712 ± 0.004
β	0.282 ± 0.005 $+5\Delta q_c$	0.285 ± 0.001 $+5\Delta q_c$	0.596 ± 0.002 $+12\Delta q_c$	0.595 ± 0.008 $+17\Delta q_c$

(*c*) *Overall estimates of* β.

$d = 2$	$d = 3$
0.28 ± 0.01	0.59 ± 0.02

† De'Bell and Essam 1983a, b.

the basis that the reasonably linear pole–residue plot had to be extrapolated from the lowest pole position at $q = 0.3570$. The central value of the overall estimate (0.28), obtained by averaging the four results, is the same as given by Blease (1977), but we feel confident in reducing the error bar from 0.02 to 0.01. If the error analysis is to be taken seriously and universality is assumed, a value of β between 0.27 and 0.28 is quite probable.

The three-dimensional values agree well with one another but the site problem results are rather lower than those for the bond problem. In obtaining the central value of the overall estimate (0.59), which is slightly higher than the average, we have attached more weight to the bond problem for which the approximants are better converged. The result represents a slight adjustment of the value (0.60) of Blease (1977) and our reduction of his error bar from 0.05 to 0.02 results from our refined critical point estimates based on longer series.

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